IAS program HKUST — 14 Jan 2019

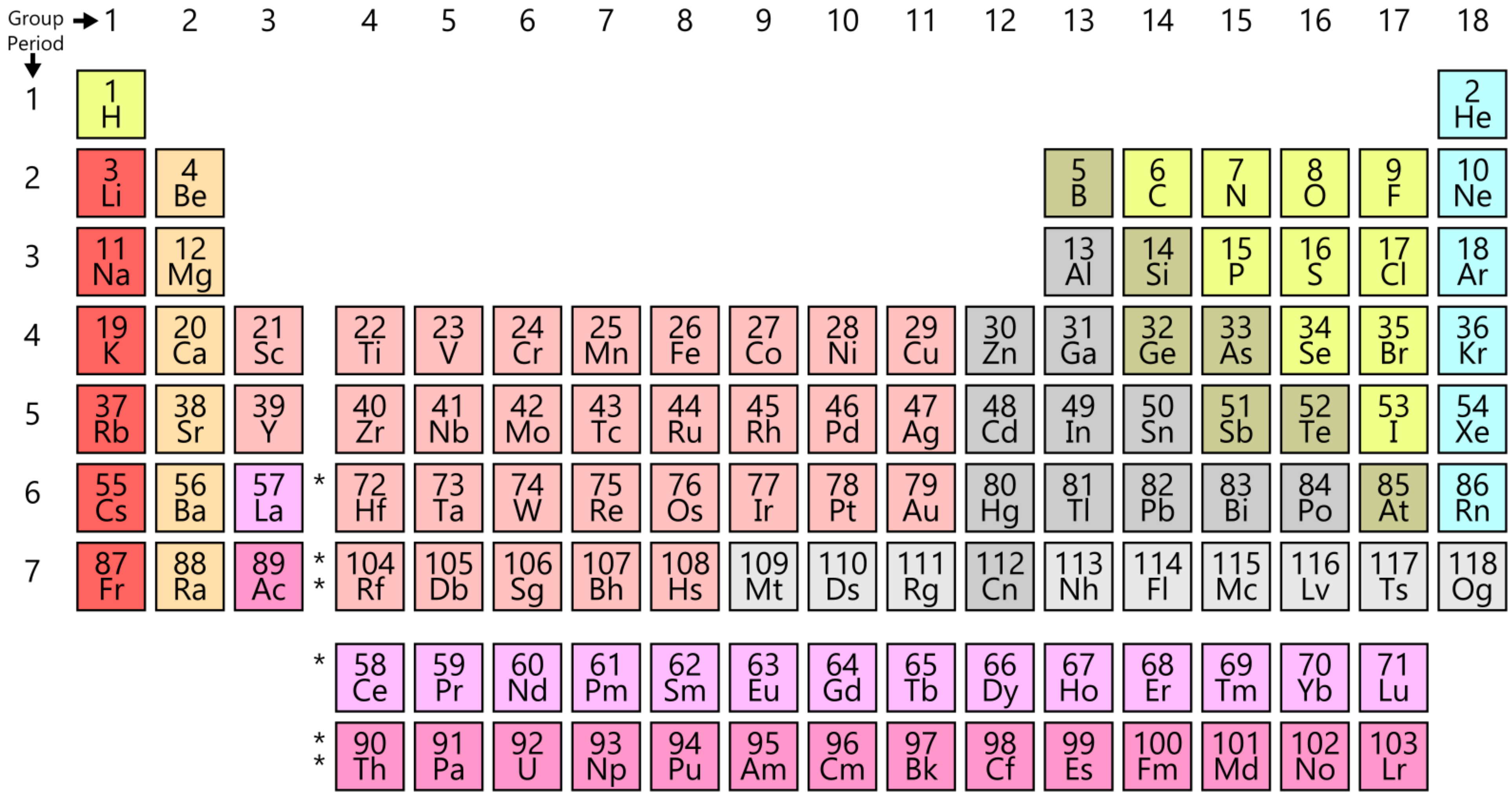
Cosmological Production of Dark Nuclei

[arXiv:1812.08784 with Michele Redi]

Andrea Tesi

INFN Istituto Nazionale di Fisica Nucleare
SEZIONE DI FIRENZE

Visible Matter



Dark Matter

χ, a stable fermion with EW charges!*

*OK, but also less minimal scenarios can be *simple* and give *interesting* dark physics

Outline

- Introduction to Baryonic Dark Matter
- Dark Big Bang Nucleosynthesis
- Example: nucleon triplet of SU(2)
- Example: nucleon singlet with dark photon
- Outlook

Introduction to Baryonic Dark Matter

Stability of Dark Matter

robust property if it is related to an accidental symmetry

$$q \to e^{i\alpha/3} q$$ in the SM, stability of matter thanks to $\mathrm{U}(1)_B$

fixed by gauge structure of the theory at renormalizable level

What's accidentally stable?

- proton

What's not?

- wino in the MSSM

how to build model based on accidental stability?

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm DM} + \sum_i \frac{\mathcal{O}_i}{M_{\rm Pl}}$$

Minimalistic approach:

[Cirelli, Strumia] χ part of an SU(2) multiplet w/o hypercharge (quintuplet)

A bit of theory bias:

- Consider new ‘dark’ sectors, with QCD-like interactions
- Just a twist on the minimalistic approach

Vector-like confinement

[Kilic, Okui, Sundrum]

$$\mathscr{L} = \mathscr{L}_{\rm SM} - \frac{\mathcal{G}^a_{\mu\nu}\mathcal{G}^{a\,\mu\nu}}{4g^2_{\rm DC}} + \bar{\Psi}(i\not{D} - m_\Psi)\Psi + y\bar{\Psi}H\Psi$$

- SU(N) gauge theory confines at scale Λ
- Valid up to the Planck scale
- Fundamental fermions have electroweak charges

We have:

$\Psi \to e^{i\alpha}\Psi$ accidental dark baryon number conservation

We also assume:

- Fermion masses much smaller than confinement scale $m \ll \Lambda, \quad \Lambda \gtrsim \mathrm{TeV}$

Spectrum of the theory

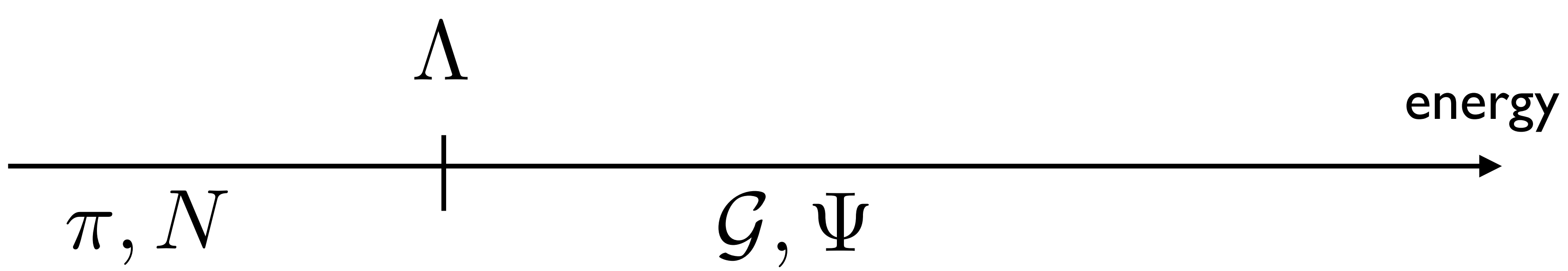
it depends on the representation dimension of the fermions

flavor symmetry as in ordinary QCD

$$\mathrm{SU}(\mathrm{N}_F)\times\mathrm{SU}(\mathrm{N}_F) \to \mathrm{SU}(\mathrm{N}_F)$$

Concretely, we will consider $N_F = 3$

- dark pion octect, decomposed under the SM
- dark baryon octect, decomposed under the SM



Dark Matter candidates in VLC

cosmological stability	dimension 5	dimension 6
dark pions	$\times$	$-$
dark baryons	$-$	$\checkmark$

Good candidates among dark baryons [Antipin, Redi, Strumia, Vigiani]

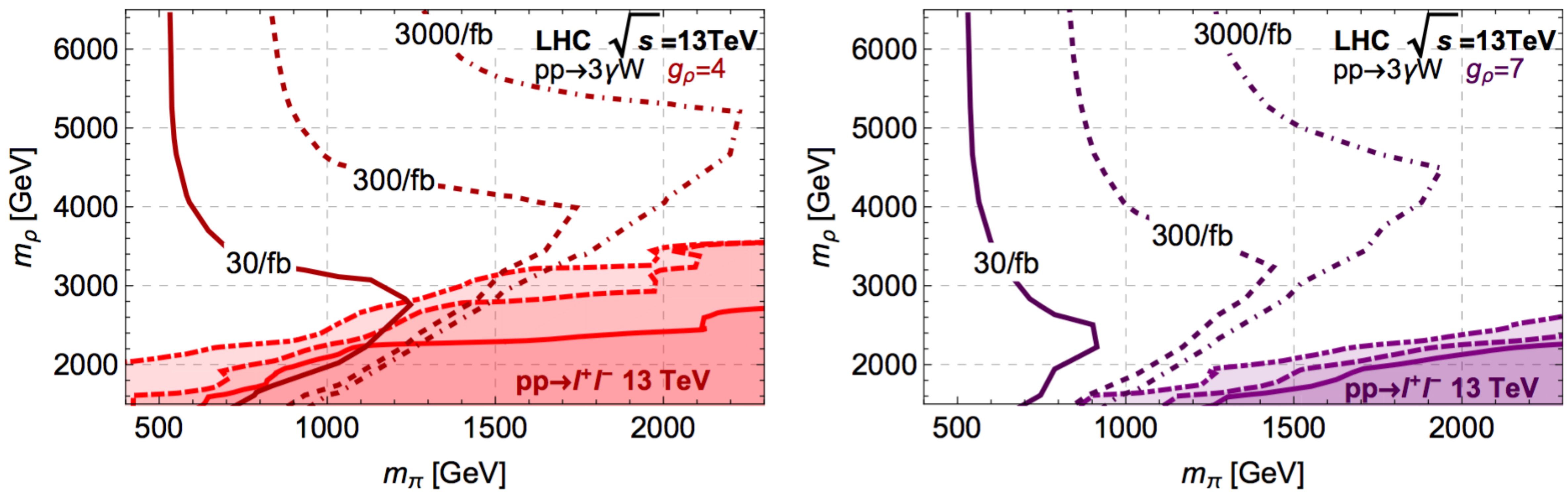
$N \sim \Psi\Psi\Psi ... \sim \mathbf{3}_0, \mathbf{5}_0, \mathbf{7}_0, ...$ under SU(2) U(1)

simplest model: Ψ triplet of SU(2)

$$N = \Psi\Psi\Psi, \quad N = \mathbf{8} \overset{\text{SM}}{\rightarrow} \mathbf{3} + \mathbf{5}$$

Constraint from the LHC

even for only weakly charged states, constraints are around TeV



[1805.12578/JHEP08(2018)017
with Barducci, De Curtis, Redi]

this sets O(TeV) scale also for Dark Matter in related models

Dark Big Bang Nucleosynthesis

[Krnjaic, Sigurdson]
[Hardy, Lasenby, March-Russell, West]
[Detmold, McCullough, Pochinsky]

Residual strong forces and binding energies

nuclear physics affected by unnaturally large scattering lenghts a [see later]

$$E_d/m_p \approx 0.0022$$

for the dark sector we consider

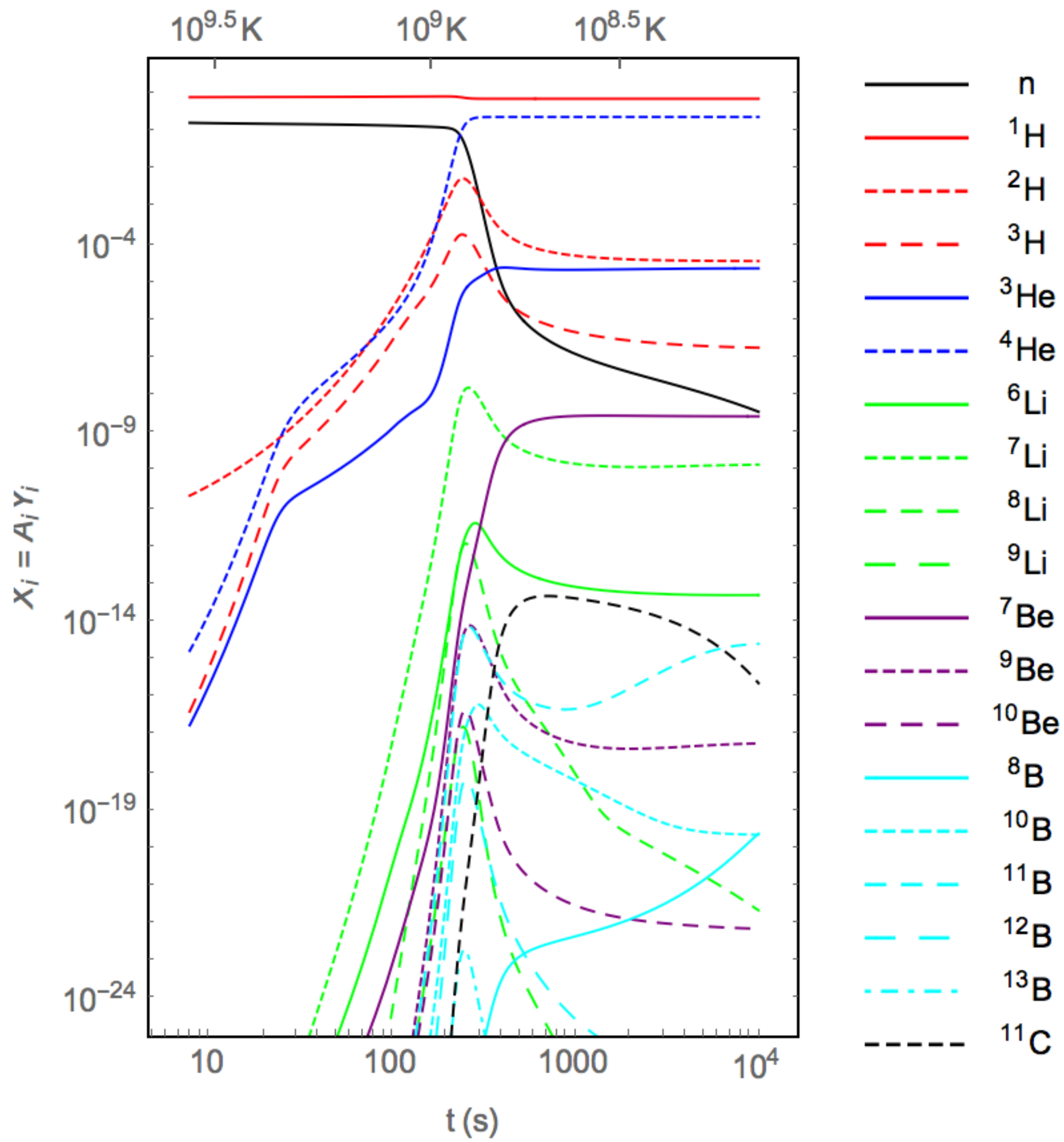
$$0.001 \lesssim \frac{E_B}{M_N} \lesssim 0.1$$

howevere, we restrict to the the scenario

- nucleons splitting $\Delta M_N \sim \frac{\alpha_2}{4\pi} M_N \quad \gtrsim \quad \Delta E_B \sim \frac{\alpha_2}{a}$ splitting of nuclei
- dominance of nuclear binding energy over weak binding $E_B \gtrsim \alpha_2^2 M_N/4$

BBN, cornerstone of cosmological history

many detailed codes, what are the structural properties?

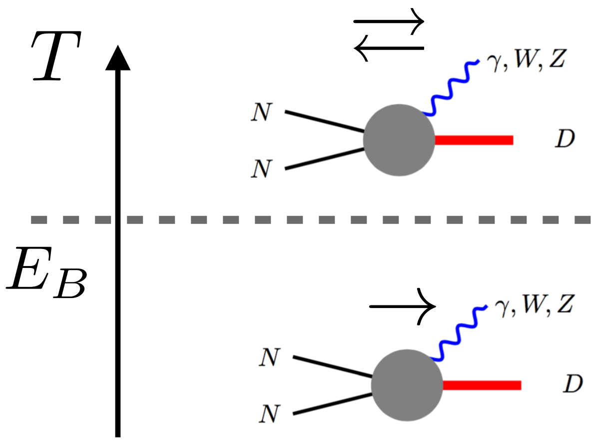


[PRIMAT]

BBN, structural and accidental facts

Structural:

- Time scale set by deuterium E_B
- Need to produce deuterium first
- Need emission of light quanta (weak process)



Accidental (typical of SM):

- Overall mass scale ~GeV
- Neutron decay
- Binding energy bottlenecks after 4He
  $\eta_b \sim 10^{-10}$

in the SM deuterium formation is at saturation

→ precise knowledge of $\sigma(NN \to D\gamma)$ **not crucial**

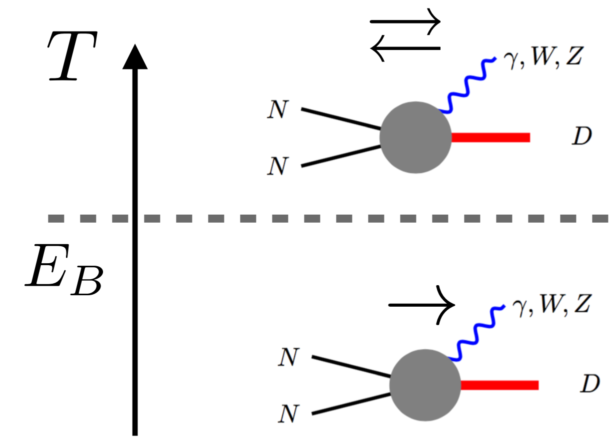
what about dark BBN?

The discussion is simplified assuming asymmetric abundance
we only track the redistribution of dark baryon number*

$$Y_{\rm DM} \approx 10^{-13}\left(\frac{\rm TeV}{M_N}\right), \qquad 1 = \sum_{A_i} \frac{A Y_{A_i}}{Y_{\rm DM}} \equiv \sum_{A_i} X_{A_i}$$

evolution of dark deuterium number

$$\dot{n}_D + 3Hn_D = \langle \sigma_D v \rangle \left[n_N^2 - \frac{(n_N^{\rm eq})^2}{n_D^{\rm eq}} n_D \right]$$



accidental facts of dark BBN

An approximate solution to the dark deuterium density

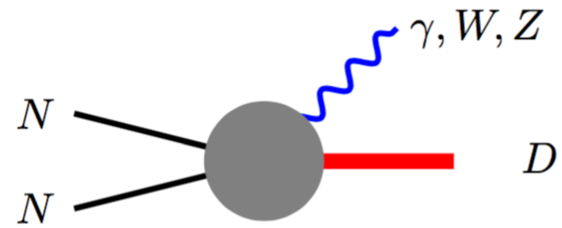
$$X_D = 5\% \left(\frac{3\mathrm{TeV}}{M}\right)^2 \left(\frac{E_B/M_N}{0.05}\right) \left(\frac{\langle \sigma_D v \rangle}{\alpha/M_N^2}\right) \left(\frac{g_\star}{106.75}\right)^{1/2} \left(\frac{25}{z_f}\right)$$

dark deuterium is rarely at saturation
(dark tritium even less abundant)

→ knowledge of $\sigma(NN \to D\gamma)$ **crucial**

Dark Deuterium production rate

precise predictions for dark BBN



exploiting the universal properties of short range nuclear forces

Theory of the Effective Range in Nuclear Scattering

H. A. Bethe
*Physics Department, Cornell University, Ithaca, New York**
(Received February 28, 1949)

The scattering of neutrons up to about 10 or 20 Mev by protons can be described by two parameters, the scattering length at zero energy, a, and the effective range, r_0. A formula (16), expressing the phase shift in terms of a and r_0 is derived; it is identical with one previously derived by Schwinger but the derivation is very much simpler. Reasons are given why the deviations from the simple formula are very small, as shown by the explicit calculations by Blatt and Jackson.

in modern language we use non-relativistic EFT for NN-scattering
[Kaplan, Savage, Wise 1996]

dark nucleon EFT

minimal interactions, magnetic dipole operator and 4-fermi interactions

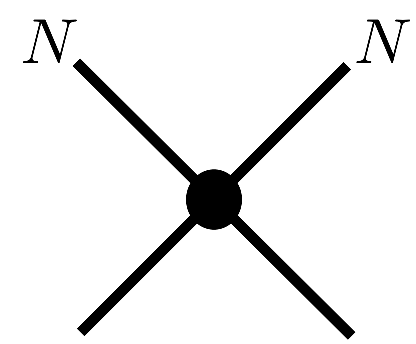
$$\mathscr{L} = N^\dagger \left(iD_t + \frac{\vec{D}^2}{2M_N} + \frac{D_t^2}{2M_N} \right) N + \mathscr{L}_4 + \frac{\kappa}{M_N} g_2 N^\dagger J^a (\vec{\sigma} \cdot \vec{B}_a) N$$

elastic scattering per spin/isospin given by

$$\mathscr{L}_4 = -\sum_{\text{channel}} \frac{C_{\text{channel}}}{4} (N P_{\text{channel}} N)^\dagger (N P_{\text{channel}} N)$$

the above lagrangian fully determine the bound state formation
under the assumption of 'shallow' bound states
let's see how…

Matching to elastic scattering



$\mathcal{A} \sim C_{\text{channel}}$

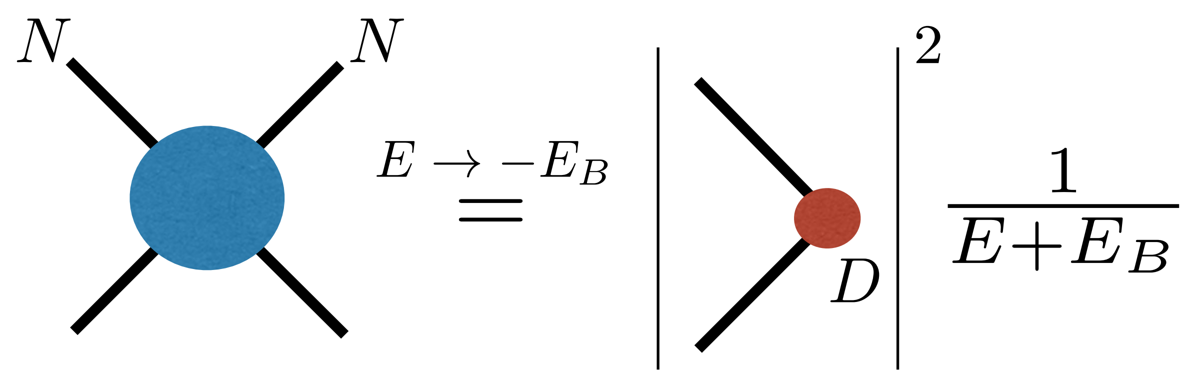
$$\mathcal{A} = \frac{4\pi}{M_N} \frac{1}{p\cot\delta - ip}, \quad p\cot\delta|_{s-wave} \approx -1/a, \quad p = \sqrt{M_N E}$$

scattering lenght	$\mathscr{L}_4$	poles of $\mathcal{A}$	bound state	SM case
$a \to 0$	irrelevant	$-$	$\times$	$-$
$a \ll 0$	relevant	$E = \|\epsilon\|$	$\times$	(nn): $a(^1S_0) = -23.75\text{fm}$
$a \gg 0$	relevant	$E = -\|\epsilon\|$	✓ $\gamma = \sqrt{M_N E_B} = 1/a$	$d(pn)$: $a(^3S_1) = 5.38\text{fm}$

effects of $\mathscr{L}_4$ need to be included at all order when $|a|$ is large

[see lectures by Iain Stewart]

Matching to elastic scattering



coupling to the bound state read off from the residue

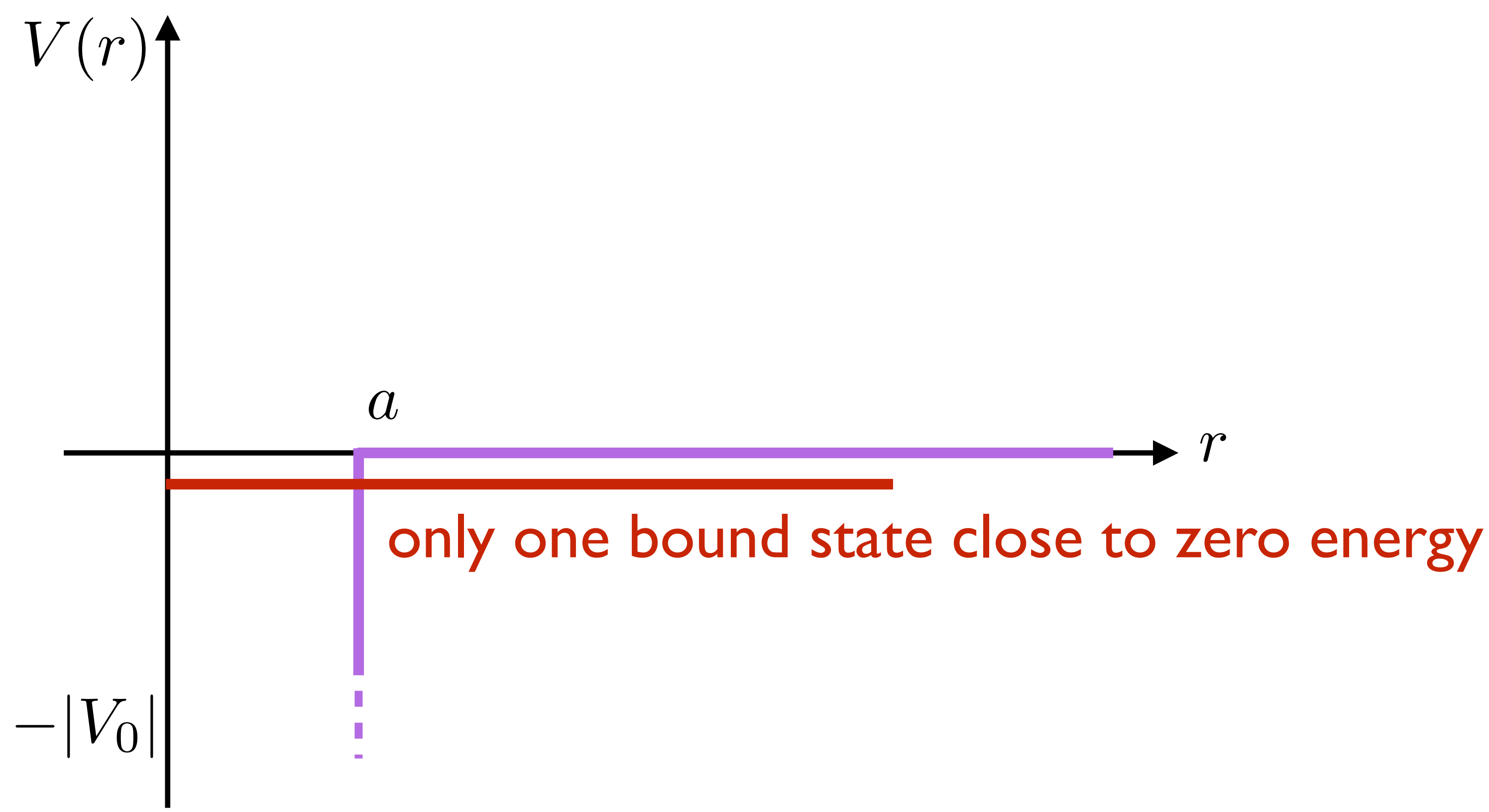
$$g_{NND} = \frac{\sqrt{8\pi\gamma}}{M_N} = \frac{\sqrt{8\pi\sqrt{M_N E_D}}}{M_N}$$

analog of the wave function overlap in Coulombian bound state formation (BSF)

by matching the EFT to the measured/given scattering lenghts/ binding energies *alone*
we *fully* determine the ingredients needed for the BSF rate

Validity of the EFT

appropriate description in presence of shallow bound states

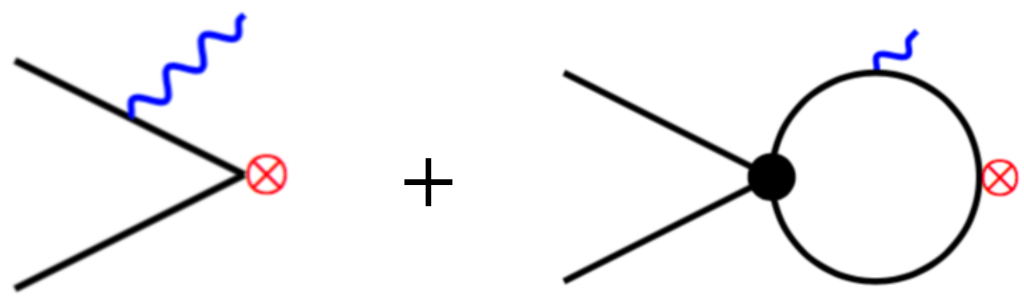


it obviously fails in the Coulombian case

[see original works by Kaplan, Savage, Wise from the '90s]

[for SM deuterium: Kaplan, Savage, Scaldeferri, Wise; Rupak]

Magnetic transitions

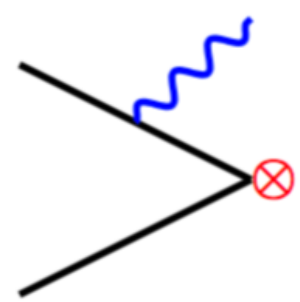


$$\sigma v_{\rm rel}\Big|_{\rm magnetic} = K_M \frac{\pi\alpha}{M_N^2}\left(\frac{E_{B_{\rm final}}}{M_N}\right)^{\frac{3}{2}}(1-a_{\rm initial}\gamma_{\rm final})^2$$

everything in terms of scattering lenght and binding energies
K, group factor calculable case by case

enhancement from initial state
in the SM $a_{(^1S_0)}\gamma \approx -5$,
slow neutrons are captured via magnetic dipole interactions

Electric transitions (aka dipole approximation)



$$\sigma v_{\rm rel}\Big|_{\rm electric} = K_E \frac{\pi\alpha}{M_N^2}\left(\frac{M_N}{E_{B_{\rm final}}}\right)^{\frac{1}{2}} v_{\rm rel}^2$$

everything in terms of scattering lenght and binding energies
K, group factor calculable case by case

no enhancement from initial state
(initial state is p-wave)

Example: nucleon triplet of SU(2)

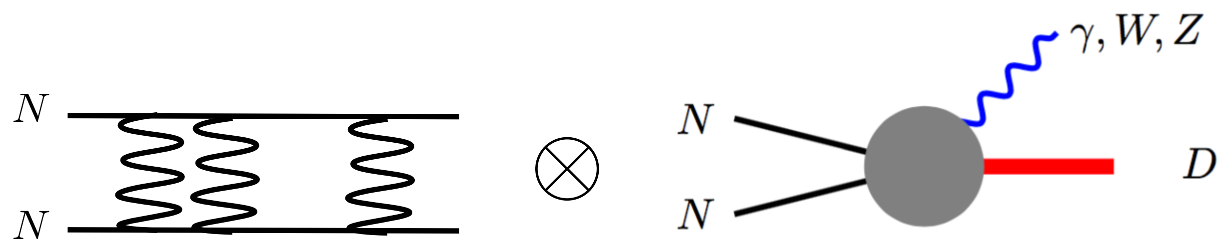
Dirac triplet of SU(2), V

dark nuclei with dark baryon number two live in the product $V \times V$

name	$\mathbf{r}$ of SU(2)	Spin
D_1	1	0
D_3	3	1
D_5	5	0

$E_{D_1} > E_{D_3}$

in the symmetric limit, absence of Coulomb barrier for singlet and triplet,
we have Sommerfeld enhancement



quintuplet is repulsive, also we do not consider bound state there

Boltzmann equation

we consider symmetric Sommerfeld effect (SU(2) preserved)

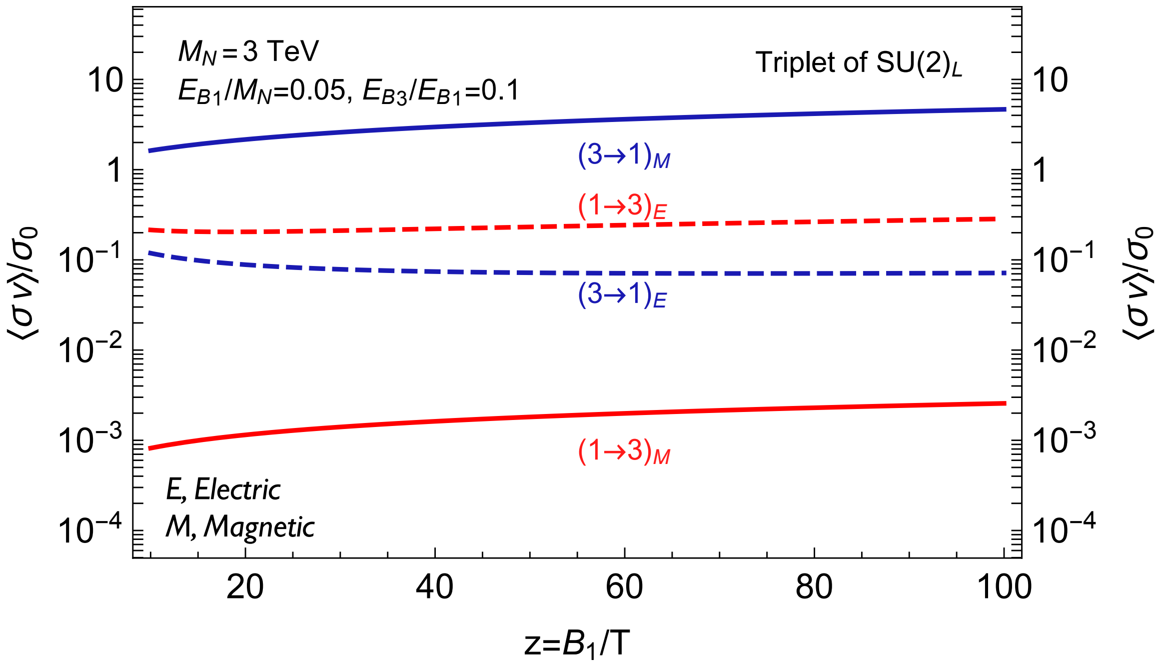
$$\frac{dX_D}{dz} = 2\frac{c\sqrt{g_\star}M_{\rm Pl}E_B\langle\sigma_D v\rangle_{\rm eff}}{z^2}Y_{\rm DM}\left[(1-X_D)^2-\beta\Big(\frac{g_N^2}{g_{eff}g_\star}\Big)\Big(\frac{z^{3/2}e^{-z}}{Y_{\rm DM}}\Big)\Big(\frac{M_N}{E_B}\Big)^{3/2}\frac{X_D}{2}\right]$$

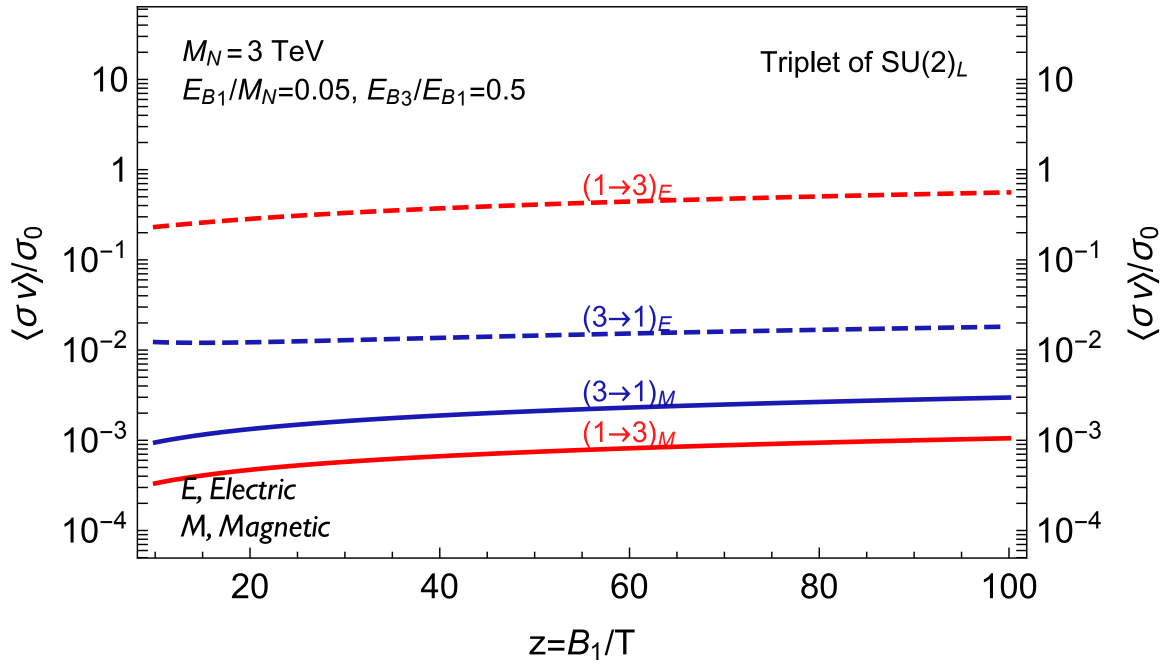
transition between bound states are faster than dissociation rates, we can sum:

$$(\sigma v_{\rm rel})^{\rm eff} = \sum_i (\sigma v_{\rm rel})_i\,, \quad g_D^{\rm eff}(T) = \sum_i g_{D^i}\exp\left[-\frac{E_{B_1}-E_{B_i}}{T}\right]$$

Cross sections

many rates contributes to $\langle \sigma v \rangle^{\rm eff}$

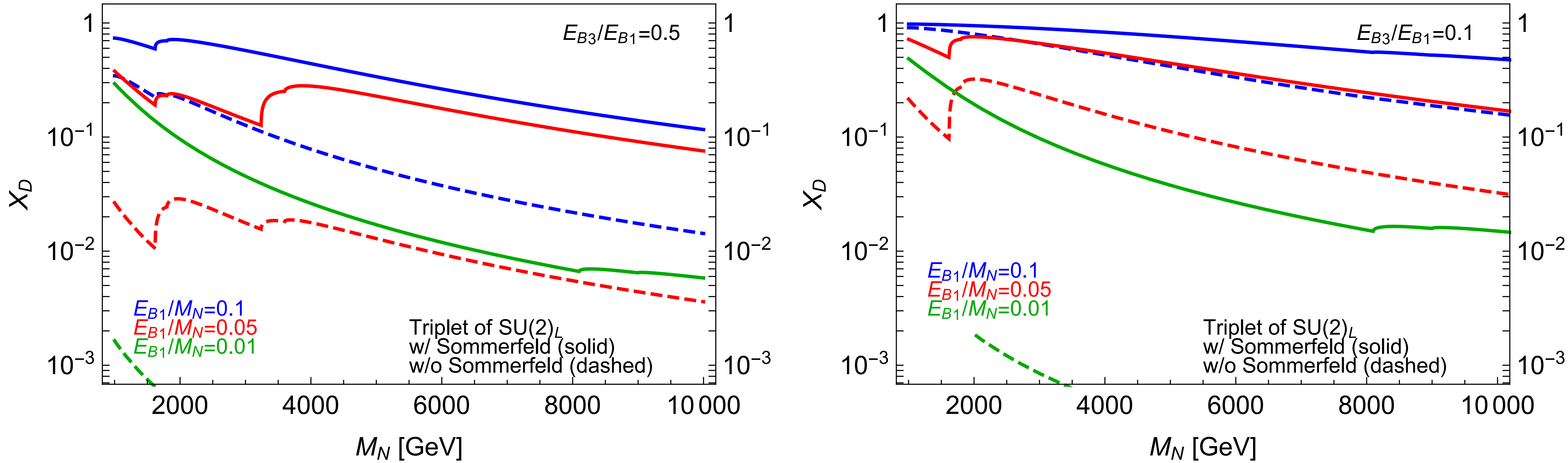




- Effect of less bind D3 can be important
- Important effect of Sommerfeld enhancement for the electric (p-wave) rates

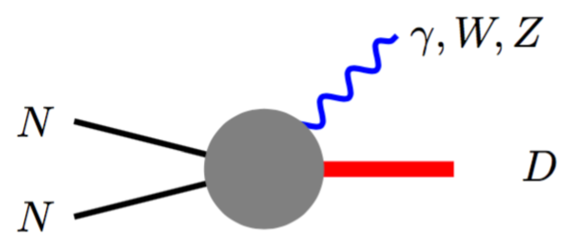
$$
\begin{aligned}
\mathrm{SE}_{s-\mathrm{wave}} &= \frac{2\pi\alpha_{\rm eff}/v_{\rm rel}}{1-e^{-2\pi\alpha_{\rm eff}/v_{\rm rel}}} \approx \frac{2\pi\alpha_{\rm eff}}{v_{\rm rel}}, \\
\mathrm{SE}_{p-\mathrm{wave}} &= \left[1+\left(\frac{\alpha_{\rm eff}}{v_{\rm rel}}\right)^2\right] \frac{2\pi\alpha_{\rm eff}/v_{\rm rel}}{1-e^{-2\pi\alpha_{\rm eff}/v_{\rm rel}}} \approx 2\pi\left(\frac{\alpha_{\rm eff}}{v_{\rm rel}}\right)^3
\end{aligned}
$$

Abundance of Dark Deuterium D_1



- Large effect only for very large binding energies
- Fraction of O(10%) recombined into dark deuterium

Indirect detection signals



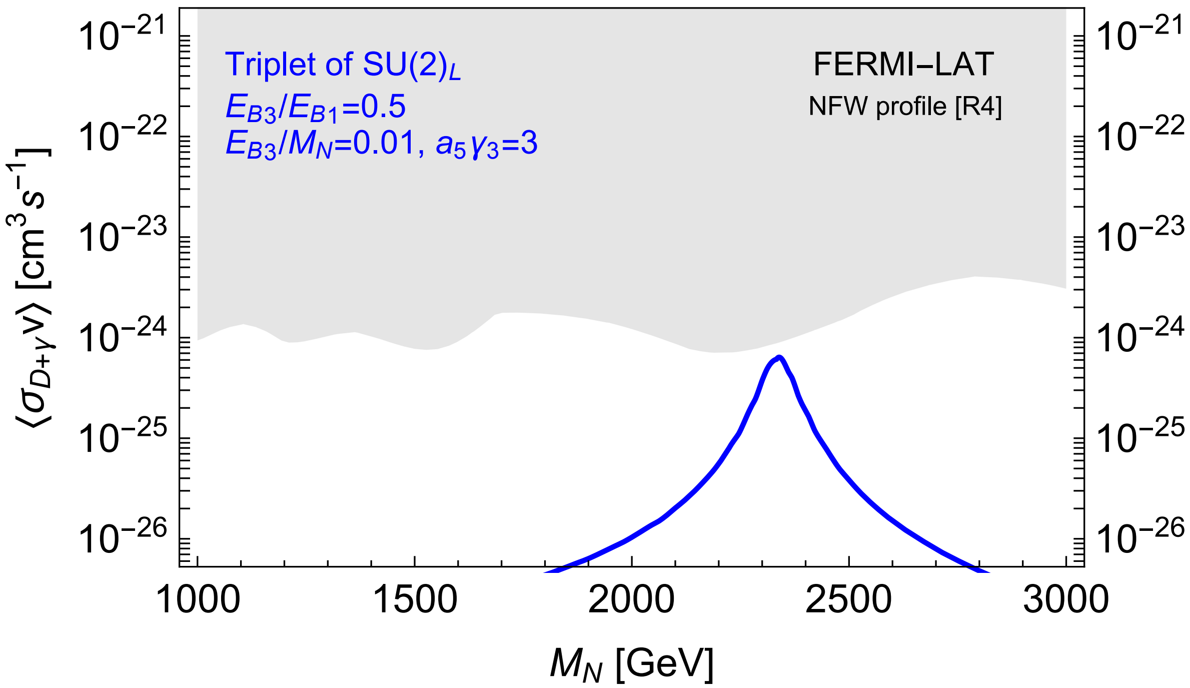
can happen today from annihilation of residual V_0
possible even in presence of totally asymmetric component

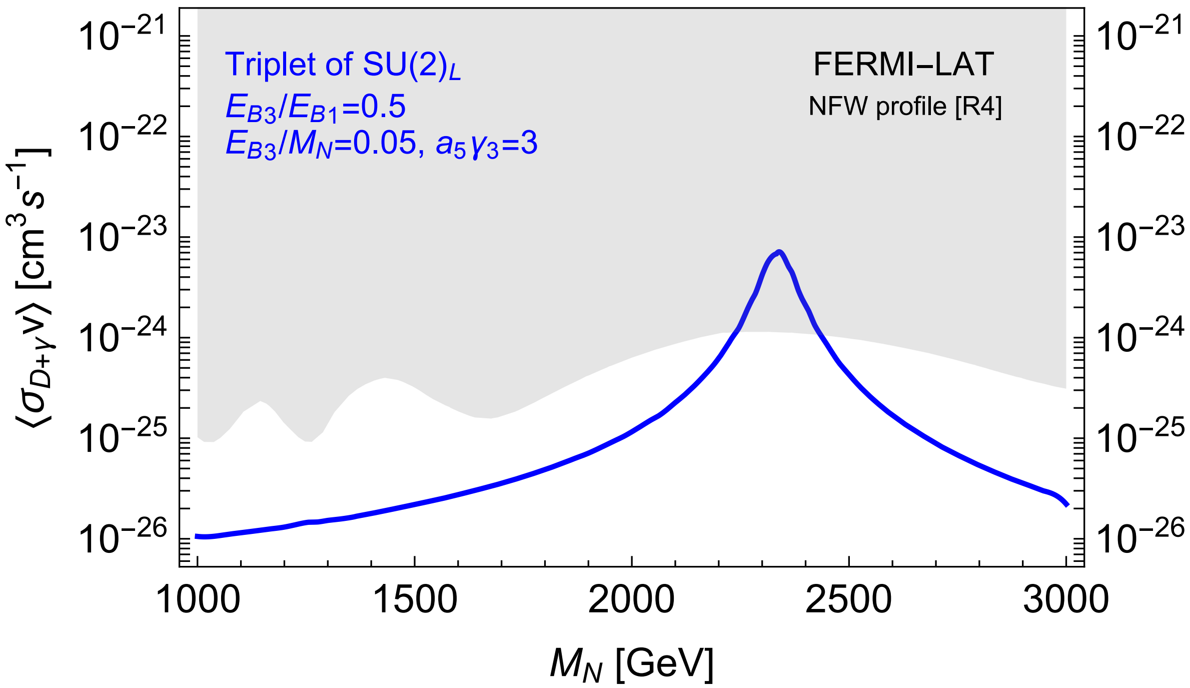
Sommerfeld effects allow for the transition:

$$|V_0 V_0\rangle^s_{S=0} \;\rightarrow\; |V_- V_+\rangle^s_{S=0} \;\rightarrow\; D_3^0 + \gamma$$

we can reuse the results for the Wino

Indirect detection signals





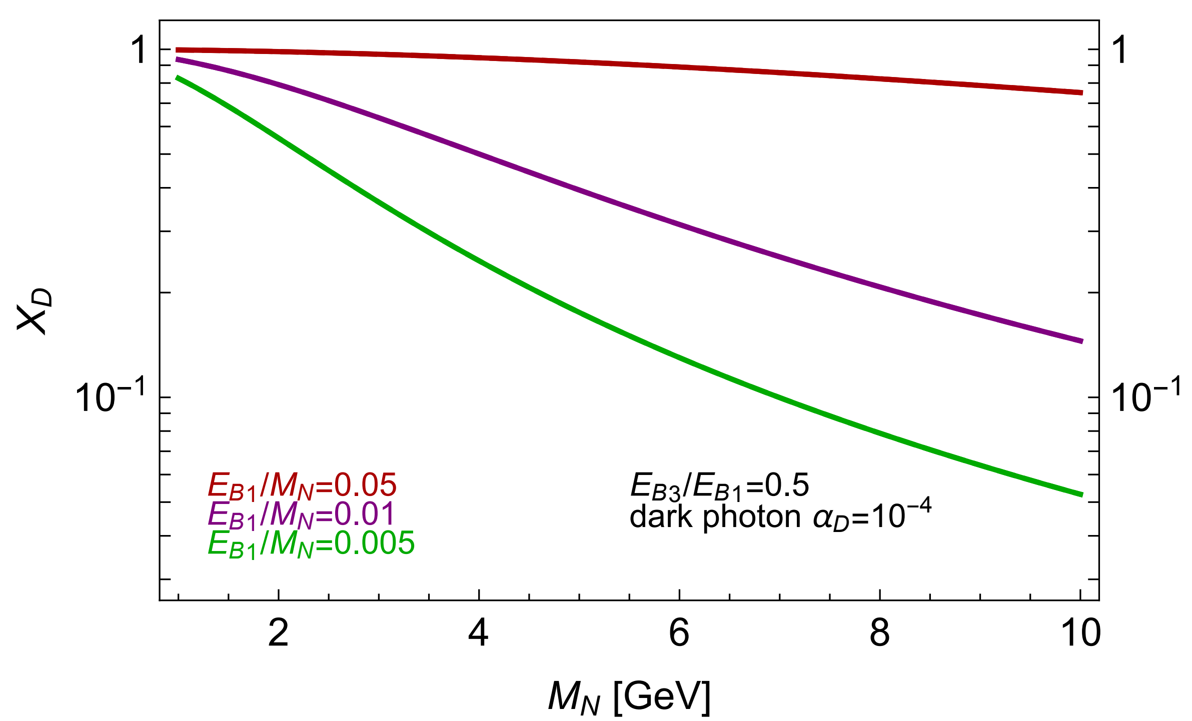
$$\sigma_{V_0V_0\to D_{\mathbf{3}}^0+\gamma} = \mathrm{SE}_{00\to+-}\left[\sigma_{V_+V_-\to D_{\mathbf{3}}^0+\gamma}\right]_{\mathrm{hard}}$$

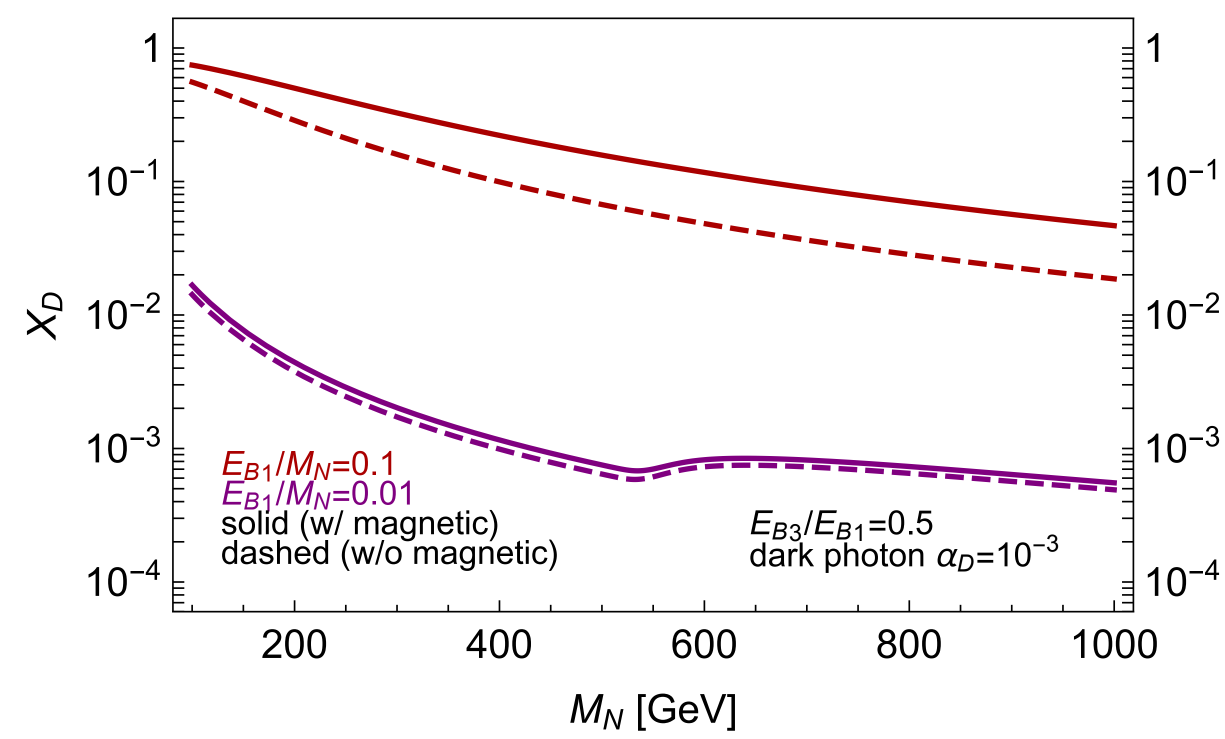
$$\mathrm{SE}_{00\to+-} = \frac{\left[\sigma_{\chi^0\chi^0\to\gamma\gamma+\frac{1}{2}\gamma Z}v_{\mathrm{rel}}\right]_{\mathrm{full}}}{\left[\sigma_{\chi_+\chi_-\to\gamma\gamma+\frac{1}{2}\gamma Z}v_{\mathrm{rel}}\right]_{\mathrm{hard}}}$$

Example: nucleon singlet with dark photon

Mimicking the SM

new Dirac fermions $Q^{\pm}$ $\quad Q \times Q = \mathbf{1}_1 + \mathbf{3}_0 \quad \rightarrow \ {}^3S_1 + {}^1S_0$





different results with previous studies [Krnjajic, Sigurdson]

input for studies that focus on large atomic numbers [Hardy et al]

Outlook

Conclusions

- Rich phase in the dark sector, if it is similar to the SM
- Dark BBN can be computed from first principles
- Improvement on existing treatments
- A strongly-coupled analogy of BSF widely studied in the literature

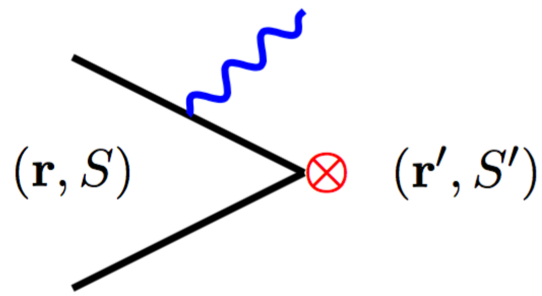
Future Directions

- Clarify better the implications for indirect detection
- Sommerfeld from strong interactions?
- Explore the dark sector in more extreme condition (large density)

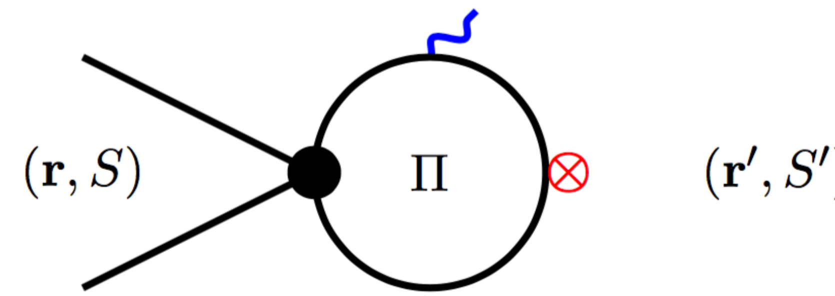
Thank You!

BACKUP
(calculation of enhancement from initial state)

$\mathcal{A}_1 =$



$\mathcal{A}_2 =$



$$
\begin{aligned}
\Pi(k) &= \int \frac{d^4q}{(2\pi)^4} \frac{i}{-q^0 - \vec{q}^2/(2M_N)} \frac{i}{q^0 - \vec{q}^2/(2M_N)} \frac{i}{q^0 - |k| - \vec{q}^2/(2M_N)} \\
&= \int \frac{d^3q}{(2\pi)^3} \frac{M_N^2}{q^4 + M_N|\vec{k}|q^2} = \frac{M_N^2}{4\pi\sqrt{M_N|\vec{k}|}} + O(E^2)\,.
\end{aligned}
$$

$$
\frac{\mathcal{A}_2}{\mathcal{A}_1} = -a_{\mathbf{r}}\gamma_{\mathbf{r}'}
$$

BACKUP
(group theory factors)

$$(\sigma v_{\rm rel})^{\rm mag}_{aMM'} = \kappa^2 \frac{2^8}{g_N^2}\sigma_0\sqrt{1-\frac{M_a^2}{E_B^2}}\left(\frac{E_B}{M_N}\right)^{\frac{3}{2}}(1-a_{\mathbf{r}}\gamma_{\mathbf{r}'})^2|C_{\mathcal{J}}^{aMM'}|^2\,, \qquad \sigma_0 \equiv \frac{\pi\alpha}{M_N^2}$$

$$(\sigma v_{\rm rel})^{\rm ele}_{aMM'} = \frac{2S+1}{g_N^2}\frac{2^6}{3}\sigma_0 v_{\rm rel}^2\sqrt{1-\frac{M_a^2}{E_B^2}}\sqrt{\frac{M_N}{E_B}}\left(1+\frac{M_a^2}{2E_B^2}\right)|C_{\mathcal{J}}^{aMM'}|^2$$

$$C_{\mathcal{J}}^{aMM'} = \frac{1}{2}\mathrm{Tr}[\mathrm{CG}_{\mathbf{r}'}^{M'}\{\mathrm{CG}_{\mathbf{r}}^{M}, J^a\}]$$

BACKUP
(dark tritium abundance)

$$
\begin{aligned}
X_D'(z) &= 2\frac{z_0}{z^2}\left[(1-X_D-X_T)^2 - \beta\frac{(\frac{M_N}{E_D})^{3/2}\, z^{3/2}e^{-z}}{g_D^{\rm eff}Y_{\rm DM}}\frac{X_D}{2} - \frac{b_1}{2}(1-X_D-X_T)X_D - \frac{b_2}{2}X_D^2\right] \\
X_T'(z) &= 3\frac{z_0}{z^2}\left[\frac{b_1}{2}(1-X_D-X_T)X_D + \frac{b_2}{4}X_D^2\right].
\end{aligned}
$$

Where we have introduced the following notation

$$
z_0 \equiv c\sqrt{g_*}M_{\rm Pl}E_D Y_{\rm DM}\langle\sigma_D v\rangle_{\rm eff}\,, \quad b_1 \equiv \frac{\langle\sigma_T v\rangle_{\rm eff}}{\langle\sigma_D v\rangle_{\rm eff}}\,, \quad b_2 \equiv \frac{\langle\sigma_T v\rangle_{\rm eff}^{\rm strong}}{\langle\sigma_D v\rangle_{\rm eff}}\,.
$$

$$
X_D \approx 2\frac{z_0}{z_f}, \quad X_T \approx \frac{3}{8}X_D^2 b_1 + \frac{1}{8}X_D^3 b_2
$$